IMAGES AND THEIR FRAMES OF REFERENCE:
A Perspective on Cognitive Development in Elementary Arithmetic

Eddie Gray
Mathematics Education Research Centre
University of Warwick
COVENTRY CV4 7AL, UK

Demetra Pitta
Psevdas Primary School
Larnaca
CYPRUS

This paper explores the relationship between children's numerical understanding and the frames of reference of their images. Embedded within psychological approaches that evoke imagery through verbal and visual cues, the study explores the different kinds of imagery identified from elementary school children's responses to such cues and links them to levels of numerical achievement. The results suggest that images with descriptive qualities manifest through specific and/or episodic references are common to both high and low achievers. However, the images of high achievers, in contrast to those of low achievers, display a spectrum of quality that have a more generic core. Such differences may have consequences for successive process–to-concept encapsulations.

INTRODUCTION

Our efforts to gain insight into why some succeed in mathematics and others fail has consciously taken a route that considers cognitive development. Whilst acknowledging the existence of a wide range of social and cultural influences on this development, (see for example Cobb, 1987; Gruszczyk-Kolczynska & Semadeni, 1988) our interest is manifest in seeking answers to the question "*What are children really doing in their heads?"*

This paper reports part of a wider study designed to investigate the ways in which different kinds of mental image may influence children's approaches to elementary arithmetic. It is not the purpose of the paper to become caught up in the format of mental image from a propositional or visual point of view. A guiding principle is that, irrespective of such format, there are other aspects of mental imagery that requires further discussion. These may prove to be important to our understanding of cognitive development and in particular to our understanding of divergence in numerical thinking.

Drawing upon work in cognitive psychology, the paper initially considers the existence of different kinds of image with particular reference to those identified by De Beni & Pazzagalia (1995). It continues by broadening the debate and illustrating how these different kinds of image may be associated with different levels of arithmetical achievement. Images children project when stimulated by single verbal and visual cues, in concrete and abstract form, suggest that those of children with lower levels of numerical achievement are of essentially of a descriptive kind. In contrast, those who are more successful project, in an 'integrated' way, a spectrum of the different kinds of mental images that reflect descriptive and relational characteristics.

THEORETICAL BACKGROUND

Setting the Scene

The concept of image has become less clear as more progress is made on the research front (Cooper, 1995) and this paper recognises that human cognition requires different representational constructs to describe it. Consequently in our context the term image may be seen as a mental reference which is a product of imaging in any modality whether it be visual, verbal, olfactory, auditory or kinesthetic.

Images are significant components of cognition and the notion of mental reference has particular relevance for the study of mathematics. Its association with a conception of 'thing' or 'object' draws our attention to a quality of abstraction that ranges from a mental analogue of a real object to a linguistic description or a symbol (see also Pavio 1986). Such a distinction would seem to be particularly relevant for cognitive development in elementary arithmetic which, it is suggested, is grounded in successive process-to-concept encapsulations (Tall (1995)

The issue for this paper is whether those who are less successful appear to have a disposition towards particular kinds of image that are qualitatively different to those projected by children who are more successful.

Different Kinds of Mental Representation

Thomas, Mulligan and Goldin (1996) have suggested that children's internal systems of representation of numbers go through a series of changes, from a semiotic one in which meaning is established through previously constructed representations, to an autonomous stage in which a new system of representation functions independently of its precursor. Pirie & Kieran (1994) indicate that a learners strong early attachments to particular dominant images can seriously influence the development of understanding The relationship between the understanding and imagery suggests that abstract imagery appears to dominate amongst relational thinkers, concrete and memory images amongst relational thinkers (Brown & Presmeg, 1993). In elementary arithmetic such differences may emerge because those who predominantly use procedures display less inclination to filter out information (Gray & Pitta, 1997). Relational thinkers appear to reject information or, to put it another way, are more able to select the information that is more relevant to a particular situation. This would suggest that the different images identified amongst children at extremes of numerical achievement have their roots in a qualitative abstraction governing the individuals active mental process of making sense of data through personal and/or impersonal involvement.

These forms of involvement have featured as some of the attributes which guide the classification of different kinds of imagery explored by Cornoldi, De Beni and Pra Baldi (1988) and De Beni & Pazzagalia (1995). The former suggest that images spontaneously evoked from a single verbal cue may be identified as *general*, *specific* and *autobiographical* in decreasing proportions. General images represent a concept

without any reference to a particular example or to specific characteristics of the item. For example, the cue ‘table’ may evoke the response “I can see a table”. Reference to a single well-defined example of the concept without reference to a specific episode characterised specific images. Autobiographic images, seen to be special cases of the ‘specific’ category enlarged to include the involvement of the self-schema, were those which involved either the subject without a precise episodic reference or objects belonging to the subject.

De Beni & Pazzagalia questioned the meaning that may be given to the autobiographic image category. They suggested that there is a distinction between images referring to a single episode in the subject’s life (*episodic–autobiographic*) and those that actually involve the subject without a precise episodic reference (*autobiographic images*).

METHOD

Within this study images are inferred from the words of subjects. Thus, two features governed the framework for the development of cues that formed the basis for an interviewee’s response. The first is embedded in psychological approaches which evoke imagery through a verbal and visual stimulus (see, for example, Stillman & Kemp, 1996). The second is that the relationship between imagery and numerical achievement should focus on two issues:

- the existence of different kinds of image identified through distinct generation processes, and the likelihood that these can be grouped into categories.
- the relationship between an emphasis on one or more of these categories and the level of numerical achievement.

Visual and verbal cues were presented to a sample of 16 children representing the extremes of numerical achievement within each of four year groups of a primary school within the UK. Numerical achievement was measured by criterion based test results available in the school and a numerical component which formed part of a larger study of which this paper is a part. The final sample had 8 ‘high achievers’ and 8 ‘low achievers’, 2 of each drawn from children aged 8 to 11.

A modified version of the defining feature approach (see, for example, Roth & Bruce, 1995) was used to gain a sense of what it is children feel is important to communicate when faced with cues in verbal form. These included concrete words and conceptual labels that had more abstract meaning. The former, seven items which denoted things that could be perceived by one of the sense modalities and had shown they could evoke images more readily than other words (see Pavio, 1969) included the cues ‘dog’, ‘table’, ‘dots’, ‘football’. The latter, more clearly associated with elementary arithmetic, had eleven items including ‘five’, ‘thirty-three’, ‘half’, ‘three-quarters’, ‘three-eighths’, ‘naught point seven five’, ‘number’ and ‘fraction’.

Sixteen cues were presented visually, nine being visual representations of verbal cues The item bank was subdivided into two sections, pictures and icons, for example, a ‘football’, ‘dots’, a ‘table’, and symbols for example, 5, 99, 3÷4, 0.75.

Each verbal and visual cue was presented with the following instructions:

Verbal/visual: What is the first thing that comes to mind when you hear the word (or see)…?

Verbal: Talk for 30 seconds about what comes in your mind when you hear the word…

Visual: Look at this, when I tell you close your eyes and put this in your mind. Talk to me for 30 seconds. Do it now.".

Children were interviewed over two separate occasions approximately 8 weeks apart. All interviews were video-recorded, linked to field notes and transcribed.

RESULTS

Classifying Responses

Children images were classified as *general* when responses indicated that they were not talking about a specific item. For example:

"It's a surface on metal or wooden sticks" (Y4+, verbal, 'table')[1]

"Part of" (Y6+, verbal, 'fraction')

The criteria for responses identified as *specific* were extended to allow for multiple examples which were qualitatively similar. For example:

"... a cheetah is one, a rabbit is one, a dog is one a cat, a Labrador, Dalmatian, owl, eagle, buzzard, etc." (Y3–, verbal, 'animal')

"Like one, two, three, four, five, six, and ten are numbers". (Y4–, verbal, 'number')

De Beni and Pazzaglia suggested that some images could be seen to be 'contextualised' since they had distinctive and relational characteristics. However, the way in which contextualised mental representation were identified as 'item specific' and 'relational', did not satisfy the clear distinctions observed in the responses of the subjects within the current study. They could be descriptive through association with a scene or a sequence of scenes or they could have a higher order quality more in tune with Skemp's (1976) notion of relational. After detailed analysis of responses it was decided that the notion of a contextual image would be better served if there were a distinction made between *episodic* and *generic* images.

Episodic kinds of image were associated with some scene or sequence of scenes and were most often narrated in continuous speech:

"Boys can kick it around and sometimes it can get lost over the field." (8–, verbal, 'ball')

"Number five. I think of a row of numbers and light shines on number five. A light goes along and stops over the number five." (Y5–, verbal, 'number')

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[1] The indicators at the end of each example provides the age of the child and their level of achievement, the phase of items and the item iteself. Thus (Y4+, verbal, 'table') indicates a 'High achieving' nine-year-old responding to the verbally presented item 'table'. Low achievers are denoted by the symbol –.

Other responses that implied the existence of a context were more fragmentary; more a collection of disconnected, seemingly arbitrary, *generic* statements. These originated from the same general concept that served as the basis for explicit relational connections. They were not descriptions of a sequential event with a clear beginning and end but more often a collection of statements that seem to have the potential to produce new ideas. Though they had a 'general' quality, the statements diverged to produce different ideas related to the item. For example:

> "Keeps you fit. An exciting game. Millions of fans. Important in every nation. Children and adults play it. Different types of football and balls." (Y4+, verbal, 'football')

> "... maths and writing. Seven… you could be doing some adding or times and the number seven might come up. Seven is also played in sport… seen on the back or shirt… has one digit… in a phone number." (Y5+, verbal, '7').

The 'autobiographic-episodic' category, which allowed for the "occurrence of a single episode in the subject's life connected to the concept" (De Beni and Pazzaglia, 1995 p. 1361), was noted infrequently but identified as follows:

> "My friend wasn't good at fractions and she had to take extra work home." (Y4+, verbal, 'fraction')

> "We have recently done reflections and they had lots of halves in them. We had to put our mirror down the side and see the rest of it. I saw lots of those." (Y4+, visual, 'half')

Though De Beni and Pazzaglia's approach requested subjects to construct 'good and vivid images' of 'high value nouns' given as cues, the current study involved reporting images of abstract nouns and of symbols or icons representing them. Such stimuli could evoke mental representations of a proceptual nature (Gray & Tall, 1994), for example:

> "It's divisible by nine" (Y6+, verbal, '99'),

> "3 parts out of 4, fraction, 0.75, more than half." (Y6+, visual '3/4')

Analysis of Results

Here we present results relating to the 30-second response that allowed children to contribute as much as they felt able to. Consequently, from each child, there may be a sequence of responses that embrace different kinds of image. All responses from each child were classified. Figures 1 and 2 display the different kinds of representations recorded as a proportion of the total number of child responses given (N). To provide a clearer sense of the more dominant mental representations classifications identified in less than 8% of instances are collated in the category 'Other'. This figure may be regarded as quite arbitrary, but careful consideration of the summarised results suggested that percentages up to this level frequently indicate more idiosyncratic and less common behaviour.

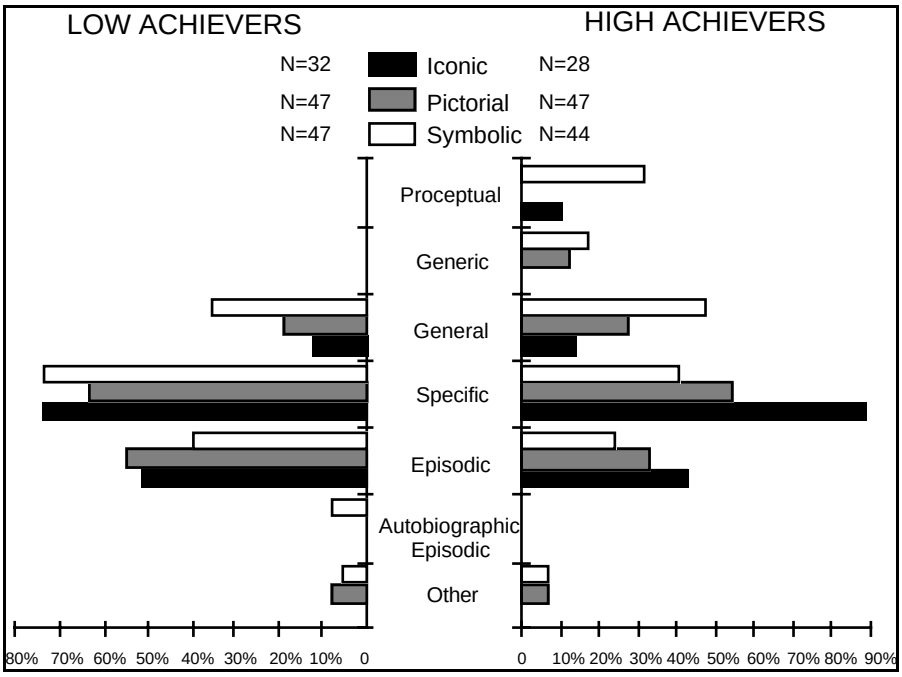

Figure 1: Occurrence of the different kinds of mental representation as a proportion of the number of items considered in the second response of the visual phase

Figure 1 provides a summary of responses to the visual items. The dominance of specific images may be clearly seen amongst the low achievers. These, together with the episodic form, are clearly dominant in reactions to each set of cues: the pictorial, iconic and symbolic.

Images from the numerical cues by high achiever's covered a wide spectrum of the different kinds. However, like the low achievers, high achievers appeared to have had difficulty detaching themselves from mental representations associated with specific and episodic content when cued by icons. These had easily distinguishable surface features but were less easy to name and connect with different experiences. It is suggested that these features militated against the abstraction of the intrinsic qualities that would have lead to the projection of generic and proceptual kinds of image.

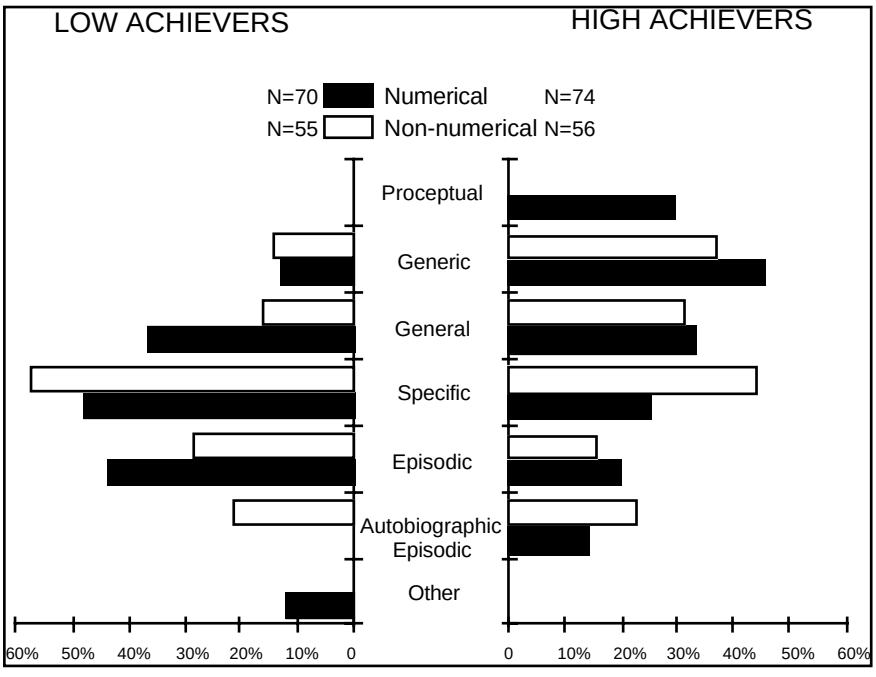

Figure 2: Occurrence of different kinds of mental representation as a percentage of the number of verbal items in the second response of the verbal phase.

Figure 2 summarises images associated with the verbal cues. The proceptual and generic images identified from high achievers when responding to these items suggest they have an ability to link the cues to different experiences so that intrinsic similarities may be abstracted. In contrast low achievers continue to project specific and episodic images but the symbols evoke a relatively high proportion of general ones.

DISCUSSION

This study supports the notion that different kinds of imagery may be identified amongst children of elementary school age. Additionally, it suggests that these may be identified through responses to both verbal and visual cues of either concrete or abstract nature and the evidence points to qualitative differences in the kinds of imagery projected by children at extremes of numerical achievement. Both groups projected imagery underpinned by descriptive qualities in that they are specific and/or episodic. However, whilst low achievers consistently project features embedded in such images, high achievers projected more relational imagery to display a spectrum of quality which, it is conjectured, has a more generic core.

Such differences, pivoting as they do around tendencies to project descriptive and/or relational images, are made apparent by the inclusion of generic and proceptual kinds of image. Perhaps it is no surprise to see that children who are selected on the basis of achievement, with the implications this may have for their proceptual/procedural interpretation of symbolism at an operational level (Gray & Tall, 1994), reflect such differences. However, the paper goes further than this. Not only do children at different levels of arithmetical achievement project qualitatively different images when prompted by numerical cues, they also project qualitatively different images of other conceptual ideas that are presented free of context through verbal and the visual cues.

It is hypothesised that the high achiever's reactions to the different phases may be accounted for by the processing differences that apply between the presentation of a visual stimulus and the presentation of a verbal stimulus. Though the invitation to consider the 'first thing that comes to mind' has not been considered in this paper, its analysis suggests that high achievers initially provide a general mental representation, often through naming or by giving a general comment about the item without any reference to other characteristics. It is conjectured that this needs to be done before a mental search to retrieve generic or proceptual qualities is carried out. When it was too difficult to project a general mental representation high achievers did not give a response but low achievers supplied a specific image associated with the surface characteristics or a specific example of the item.

Age differences have not featured in the analysis as presented. However, it does not appear to have much influence upon the quality of mental representations projected by low achievers. However, those of high achievers, being of a more generic nature, operate at a more relational level that seems to grow in complexity.

CONCLUSION

The results would seem to have important implications for our understanding of the way in which children view the development and use of numerical activity in the context of repeated process-to-concept encapsulations. The limited spectrum of mental representations projected by low achievers suggest they are either unable to, or simply choose not to, see through actions and objects to embrace more abstract qualities. It may even be that early teaching has influenced their focus of attention.

The development of elementary number requires an ability to concentrate the mind and give careful thought to an act or idea — to filter out irrelevancies and separate notions from their context. It involves the construction of relationships between and amongst objects and of the actions on them. It would seem that such a process might work to the advantage of the high achievers. Their disposition towards the formation of images that integrates descriptive and relational characteristics seems to ensure the construction of number concepts through the synthesis of pseudo-empirical and reflective abstraction Tall (1995). It is conjectured that this follows a very different cognitive development from that of children whose disposition towards descriptive images arises from their concentration upon empirical abstractions and direction, through teaching, towards the pseudo empirical.

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