# Misguided notions of mathematical development.

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## Abstract

*Judged against criteria of achievement for specified levels of attainment common to all learners, relative success is seen as a stepping stone to the next stage of development. This paper suggests that though a single ladder of achievement may be good enough to permit broad measures of assessment, it may not take into account the development of qualitative differences in thinking processes which are less easily measured. It suggests that the notion of a single ladder of development based upon the principle that children "go through the same stages at a different pace" is misguided.*

## Introduction

Short term memory plays a crucial part in the actions carried out by individuals. Only through being processed in working memory can information from the sensory part of the system enter a person's long term memory. It is only when information is called out of the person's long term memory can stored information be used in the course of thinking. The most important aspect of short term memory is its limited capacity.

Although it does not appear possible to extend the capacity of short term memory it does seem possible to aid retention so that working memory is able to cope with complex situations. One method, almost routine in many schools is to practice a procedure until it becomes routinised to the point that it may be applied in particular situations with a degree of automacity which does not require much conscious thought.

Formal markers may be laid down to indicate levels of attainment in skills and concepts and these markers are frequently based upon the notion that all children can achieve them. Standard assessment procedures may give a notion of the level of achievement of the individual. However, those who see mathematics as a collage of unrelated inflexible procedures, achieve their success with great effort without paving the long-term path to flexible success. Those who are able to focus on what is important at the time, be it process or

known concept, develop a more flexible form of mathematics that is more likely lead to success (Gray & Tall, 1994).

**The Mathematics Curriculum**

One of the problems with the formulation of a mathematics curriculum is that it is conceived by people who are successful. These are people who are able to think in sophisticated ways because of their ability to compress knowledge. Their mathematics is seen from a mature viewpoint in which the structures have great richness and interiority and they therefore have perceptions of simplicity in which this structural richness plays an implicit fundamental role.

Learners do not yet have this conceptual richness. The belief is that as long as teachers explain how to manipulate sophisticated thoughts they are giving learners power and strength. Unfortunately, more frequently than not, short term need in the form of procedural growth tends to take precedence over long term development. Failure becomes a more distinct possibility than long term success.

The human brain works in a massive simultaneous-processing manner coping with complexity by filtering out most of the activity and only focusing on a small quantity of mental data (Crick, 1994, p. 61). It therefore benefits from representations which compress information in a way that can be handled easily by the short-term focus of attention. The encapsulation of long counting processes as immediately available number concepts compresses knowledge into a form that is easily usable by the human brain. The child who is less successful at compressing processes to concepts has a harder task to achieve – a harder task for an already stressed cognitive structure.

How unfair mathematics is! It becomes *easier* for the gifted and *harder* for the less successful. Almost certainly, this is in direct contradiction to the principles underlying a common curriculum where ladders of “levels” are placed in each subject for the growing child to climb. The principle appears to be based on the democratic idea that “all children are equal” and “go through the same stages, but possibly at a different pace”.

The theory of procepts suggests that this is a totally inadequate view. It predicts that, in subjects such as arithmetic and algebra where processes are successively encapsulated as concepts using symbols, children who get stuck at the “process” level have an enormous task before them to make any sense of

successive levels. At this stage they are *not* doing the same thing as their more successful peers who are mentally handling symbols as both processes to *do* and concepts to *think*. The children who do not encapsulate do not have any meaningful concepts to think with (Linchevski & Sfard, 1991).

This suggests that the idea of a single ladder for all is misguided (Foster & Tall, 1996). It is a notion which *appears* to explain approximately what happens and seems a good enough approximation to allow broad measurements to be made to measure children's progress up the ladder. As a statistical guide to progress, discrepancies in data between different children are not regarded as important and it proves to offer a simplistic indicator of relative success of children taught by different teachers in a different schools. But there are *very significant* qualitative differences developing in thinking processes which are less easily measured and these are having very serious effects on the learning of our children.

For instance, children who do not see arithmetic symbols as flexible processes and concepts are hardly likely to see algebraic symbols in a flexible light. Therefore algebra will be meaningless to them other than a task to rote-learn procedures to get answers. The remainder of the "algebra ladder" is therefore a *different ladder* for them from that seen by the flexible thinkers and is likely to be a highly inappropriate ladder to climb.

Conversely, the single ladder also militates against the learning of more successful children who think in a different way and are being denied access to topics which are too difficult for others. For instance, the concept of fraction arises through compressing the process of sharing and is therefore a *procept.* It is typical of an idea which is impossible for those who fail to encapsulate whole number processes as concepts yet straightforward to those who have a flexible "proceptual" view of number as process and concept.

Fractions appear to be vanishing from the school curriculum because they are not democratically possible for all on a single ladder. Algebra is also "too difficult" for the majority and so insufficient exposure is being offered to those who have flexible thinking processes that are more suited to handling the symbolism.

Although simple arithmetic may be dealt with relatively successfully through the application of rules committed to memory, within an algebraic context an extensive reliance on memorising rules and procedures may prove so severely

limiting that it leads to great difficulty and eventual failure. In a procedural context the ever increasing complexity of unfolding algebraic expressions will mean that the simplification of complex expressions may prove unmanageable.

## References


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