EMILY AND THE SUPERCALCULATOR

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This paper reviews a short learning programme devised for a low achiever in elementary arithmetic. Using a graphic calculator, the programme was designed to change the quality of imagery associated with numerical symbolism. Earlier observations had shown that the child's symbolic images were episodic and active, representing mental procedures that were analogues of physical ones. By providing an alternative, non-counting dependent procedure, it was hypothesised that the calculator would encourage the formation of semantic and generic images which used symbols as objects of thought. Positive indications suggest that continuing to encourage most low achievers to count when they experience difficulty in elementary arithmetic may need reappraisal.

Introduction

Symbolism has the power to dually and ambiguously represent computational procedures and the results of these procedures (Gray & Tall, 1994). To benefit from the flexibility provided by such ambiguity the young child's conception of arithmetic must progress through several phases of compression: lengthy counting procedures which are interpretations of processes to *do* must eventually become concepts to *know*. It is through procedural compression that symbols may become objects of thought.

This is a story of one eight year old whose efforts to progress through stages of procedural compression had not provided her with the flexibility to use the power of symbols. It was hypothesised that if the 'procedural clutter' associated with the perceptual and figural items that dominated her interpretation of mathematical symbols could be removed, she too may focus on the power of symbols. To do this we provided a graphic calculator, the 'supercalculator'. The paper adds a further dimension to notions that the use of calculators not only does not harm computational ability but supports concept development (Shuard, Walsh, Goodwin and Worcester, 1991; Shumway, 1990). Research on the use of graphic calculators (Ruthven, 1993; Dunham & Dick, 1994) had indicated that there was potential for this resource within the classroom although the outcomes did not always give positive results (Ruthven, 1995). We consider the changes in the child's use of symbolism during a period using the calculator. Our focus is the opportunity that the resource may give for stimulating the construction of mental imagery associated directly with arithmetical symbols as opposed to imagery that is an analogical transformation of them.

What can imagery tell us about success and failure in arithmetic?

Pitta and Gray (1997) describe how children at extreme levels of achievement in elementary arithmetic focus on imagery which is of different qualities. Imagery identified by 'high achievers' tended to be symbolic, used to support the production of known facts and/or numeric transformations which produce derived facts. Imagery reported by 'low achievers' was usually based on analogical representations of physical objects. These images appear to be clear imitations of actions that could have taken place with real objects. Pitta & Gray went on to suggest that the essential differences between the imagery of 'high achievers' and that of 'low achievers' was that the imagery of the former was semantic and generic whilst that of the latter was episodic and active. The terms 'episodic' and 'semantic' were used to draw a distinction between those images that arise from memory associated with the recollection of personal happenings and events, and those images linked to organised knowledge associated with meaning and relationships but independent of an event. Such distinctions lead to the conjecture that the images of 'low achievers' are essential to thought. In contrast, those of 'high achievers' appear to act as thought generators. They 'flash' as memory reminders, momentarily coming to the fore so that new actions or transformations may take place. In the belief that the former is a factor of the procedural thinking associated with the proceptual divide, the issue for this paper is whether an alternative 'procedure' may discourage a 'low achiever's' need to use manipulatives in the mind but stimulate the creation and construction of symbolic images that help to generate thought.

An Alternative Procedure: Focusing on Symbols

There is a tendency within pedagogy to provide practice to confirm "understanding". For children who have difficulty with elementary arithmetic such practice is usually based upon the use of counting. It is suggested that such experience may confirm the understanding that arithmetical symbols can be transformed into physical objects, or mental analogues of these objects. These then form the basis of counting procedures. It would seem reasonable that if the learner puts effort into this solution to problems it is perhaps the case that the more procedures are remembered and the more likely they are to be used but, paradoxically, the learner may possess less understanding. However, it has been recognised for some time that calculators can give children an insight into numerical patterns (Shuard *et al.* 1991). To identify relationships between numbers children need no longer be constrained by the use of lengthy counting procedures. The supercalculator seems to have an added advantage. Combinations can be recorded and displayed in their entirety, equivalent outcomes from different procedures may also be seen at the same time (Ruthven, 1993) and the child can control the form of display on the screen. Additionally, for our attempt to minimise a focus on counting the supercalculator offered two strengths; it provided an alternative procedure which had the potential to provide an alternative representation for numbers, and it could display all symbols and operations at the same time. It was conjectured that this would offer firstly an opportunity to concentrate on numerical symbols as objects of thought, and secondly provide a stimulus which would support mental organisation. It had the potential to support the creation and use of symbolic images. It did not support analogical transformations of them.

A calculator provides and opportunity to create a number by pressing a button. It also permits a particular number to be created using the combination of a composite series of button pressing. Thus, by asking the child to create 9, this could be done by pressing 4+5=, by pressing 6+3= or it could be formed from 2+3+4 or 13–4 etc. By eliminating a counting procedure the 'alternative' procedure had the potential to create a "wholeness" about number. This may be seen at two levels; a specific one in which the focus could be on number triples, and a more generic one during which it is possible to identify the relationships between numbers and simple operations. It is unfortunately the case that many "low achievers" find it hard to switch from harder to easier methods if the first is habitual and unfamiliar (Krutetskii, 1976; Steinberg, 1985). The "button pressing" procedure had the potenial to overcome this difficulty since the child may not regard it as a mathematical activity which should become a focus of attention.

Emily

We first met Emily in February 1995. She had considerable difficulty with elementary arithmetic. Articulate and highly motivated, she was identified as one of the lowest achievers within her year group of 119 children. Test results (SEAC, 1994) placed her amongst the bottom four children. Our initial conversations with her were about the numbers 1 to 10. Her responses were dominated by descriptions of images that were analogues of physical objects. Over a series of four interviews, during which she was given elementary addition and subtraction combinations and asked to talk about her approach to each one she indicated that she relied extensively on active mental images. As the items began to involve combinations greater than ten Emily made considerable use of her fingers. She was representative of the group of low achievers who concretise symbols and focus on mental or physical manipulation (Gray & Pitta, 1996).

Verbal and written symbols of the numbers one to six were seen as mental arrays of dots in the mind. Those between seven and ten were mental images of fingers arranged in a linear fashion. Emily manipulated her mental images of dots, her preferred image, relatively easily. The solution to 4–3 was explained as:

> As I see it there's two dots above each other and then there's.... the first one, the one below and the one next to it are being taken away and there is only one left up at the top. *(Emily, 1995)*

It may well be that extensive experience with board games provided the episodic background for this frame of images:

> *When I was young, when it was winter, we often played board games because we were not allowed outside. We were using dice. We were playing all of the time using dice.* *(Emily, 1995)*

She recognised that there was greater difficulty associated with finger like images. Using these meant doing two things at once, counting and concentrating on the sequence in which each finger was used:

> *I am trying to think out the answer as well as use all of my fingers–this is confusing... with the dots it is easier [than with fingers] because you don't have to keep thinking, 'No it's that one I need to move, no, its that one, or that one... [with the dots] it doesn't matter which you move.* *(Emily, 1995)*

It was as if Emily recognised that if she used fingers she had to count particular fingers, whereas by using mental images of the dots she could use any dots. For relatively more difficult combinations such as 'nine take away six' Emily used her fingers in an indirect way by 'feeling' them without looking at them, touching them or moving them. This was no surprise since evidence had shown that it is more likely that an individual will move from a mental episode to a real episode as things become more difficult (Pitta & Gray, 1997). But this too caused problems:

> *I find it easier not to do it with my fingers at times because sometimes I get into a big muddle with them because I find it much harder to add up because I am not concentrating on the sum. I am concentrating on getting my fingers right...which takes a while. I can take longer to work out the sum than it does to work out the sum in my head.* *(Emily, 1995)*

But there was a third problem for Emily. Her perception was that any procedure that was not overt could place her in a position of conflict with the teacher:

> *If we don't [use our fingers] the teacher is going to think, 'why isn't she using her fingers–it is meant to be the easiest way–and they are just sitting there thinking. It is like, ...because we are thinking that...we are meant to be using our fingers because it is easier....which it is* ***not****.* *(Emily, 1995)*

Unlike most of the other children who formed part of the study into children's use of imagery (Pitta & Gray 1996, 1997) Emily appeared to recognise that there was a qualitatively difference between using perceptual items and mental representations of these items. It was not only that she believed the later was easier but to her it also made a difference between 'doing' arithmetic and 'thinking' about arithmetic:

> *I try not to use my hands much... I don't bother looking because I am too busy thinking so… when I am not using my hands I am trying to work the sum out.* *(Emily, 1995)*

Emily appeared to have come to some conclusions. First, it was easier to do the sum in her head and secondly, some images were better than others. It seemed to her that it was easier to see a number and remember it if it was recognised by some form of pattern like the array on a die. It was harder to think about if the representation was based upon a line of finger like objects, each being focused upon at a separate point in the counting procedure. Thirdly, arithmetic involved being seen to be 'doing', but this was unsettling because she was trying to 'think'. Unfortunately however, she was not thinking with the tools her more able peers were using, the arithmetical symbols. Her tools were analogical images of real objects manipulated in accordance with her recollections of former experiences. Numerical symbols were concretised to form objects which supported the use of mental imagery that was episodic and active. Her focus was on an action which could be simplified by the nature of the representation that she gave to the objects. However, whether or not she used dots, fingers or finger like objects the intrinsic quality of the object did not change. Her perception of quantity represented by the symbols influenced her choice of objects and the way the objects were used, so the focus turned to the nature of the action. Though it was evident that her procedural competence was sound it had not supported the encapsulation of numerical processes into concepts. She was not filtering out unnecessary information and making the cognitive shift that would lead to the realisation that symbols could become objects of thought. The longer term prognosis was that the qualitative difference between Emily's thinking and that of her more able peers would widen into a gulf.

A Programme with the Supercalculator

Emily was introduced to the supercalculator after the first series of interviews. Directed work with it extended over a period of three months, April 1995 to July 1995. The programme build around its use was not seen as simply another way of doing things. The calculator was not a means for completing the result of arithmetical combinations but a way of seeking different combinations that made a particular number. Thus she started with the number and considered different routes to it. Four phases were established to support the development:

Working with nine......

9

Making nine **1.** ....

2. .... **3.** ....

4. .... **5.** ....

Working with the calculator

Ways to make nine

1. .... **2.** ....

3. .... **4.** ....

5. .... **6.** ....

Ways to make nine starting with 5

1. .... **2.** ....

3. .... **4.** ....

5. .... **6.** ....

Ways to make nine starting with 10

1. .... **2.** ....

3. .... **4.** ....

5. .... **6.** ....

An interesting thing I have discovered..........

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1. Emily was given an opportunity to think about numbers without using the calculator
2. During this phase Emily used the calculator to support her thinking not simply to check answers. She could control the form of the numbers and seeing one combination maintained in display could try the same numbers in a different procedure. Memory usually associated with holding quantities and carrying out procedures could be directed towards thinking about number combinations.
3. At the end of each activity she consider interesting things that had been discovered during the activity.
4. She was given and opportunity to talk about individual numerals and associated combinations.

To accompany her work a specially personalised booklet was designed with each page following a pattern similar to that in the adjacent figure.

The programme called for Emily to try to complete a page of her booklet each week. Each week she discussed her work with the programme designers. During this time she was asked to talk about her numbers without access to the calculator or to her written responses.

Programme Development

Initially Emily had to overcome some reluctance to use the calculator. This stemmed largely from her perception of what others may think. However, by the end of the first week she had established that there were many ways in which she could make nine, the first number in the booklet. There were of course standard addition combinations such as 4 + 5, 3 + 6 etc. but she also provided others, 4 + 4 +1, 3 + 4 + 2, and using the starting points of 5 and 10 she now provided solutions such as, 5+1+1+2, 5 + 5 – 1, 5 + 6 – 2, 10 – 1. Emily admitted that she wouldn't have thought of these sorts of combinations earlier but her outstanding discovery for the week was that she had found out that she could add larger numbers and then take away.

> *I didn't know that you could add larger numbers and then take away. I didn't know you could go up and down.* *Emily, 1995*

As she worked through the programme written evidence of Emily's use of standard triples during the non-calculator phase tended to decline. It became noticeable that for the first four numerals in her sequence, 9, 7, 8, and 6 she gave at most two but then she provided other 'non standard' combinations. When working with 7 for example she provided 10+10+10–20–3, with eight she provided 99–91 and 34–32+6. Working with the calculator she provided written evidence of combinations such as 90–80–4=6, 2+9+1–6=6, 30–15–9=6, 40–30–5=5, 10+30–30–2=8, 5+20–19=6.

Ruthven's (1993) suggestion that different rules established through the use of the super-calculator could provide a highly motivating context for discussion formed the basis for the interviews that followed Emily's written work. This discussion provided a platform for the necessary stages of reflection. Her use of the calculator not only removed the need to focus on counting procedures but also provided an opportunity to see different descriptions of addition and subtraction procedures leading to the same results. Furthermore, from the interviews it became evident that Emily's understanding of the relationship between numbers was beginning to change.

> *Well,... before I would have found it harder with nine, but...um...its not that hard because I know that ten is really easy so nine is really easy because you just take away one from ten...* *(Emily)*

In contrast to her earlier comments in which she had indicated that she found subtraction difficult, Emily was now beginning to see a different framework for working with numbers:

> *It was easier to take away from eight than I thought it would be. Before I found it a bit hard with the other numbers. I thought eight would be a bit hard. But in the end it wasn't as hard as I thought it would be.* *(Emily)*

I have discovered it is much easier to use multiplication in sums (Emily, 1995)

Inevitably pattern became a feature of Emily's discovery. When talking about 8 the following exchange took place:

Int. *What about 30–22.*
Emily. *Well, 20–12, add another ten to twenty, then if you take away, instead of twelve… it can't be twelve, because that is much too low to take away from 30. So, I would have thought it would have been one of the twenty's, so if it was twelve it would be 22.*
Int. *Let me give you one to do now. If you started at 40 how would you make eight?*
Emily: *It would be....take away....32.*
Int: *If you started at 50....*
Emily *...take away 42.*
Int: *....and 60?*
Emily: *52.*
Int. *Why has it all become so easy all of a sudden.*
Emily: *Well, it wasn't very easy when I did the first one here, but then, if it was 40 it would be 32, and then it would be 42 and then 52.....*

By this time it was common for Emily's written work to extensively include any numbers up to 100 and at times she included numbers over 100 in her combinations. She was beginning to realise that:

It is a lot easier to work with big numbers than I thought... I thought that big numbers would be very hard because they are so big... but it isn't. It is just the same as low numbers. (Emily, 1995)

It was evident from our discussions that Emily was now talking about numbers as objects. During all of the interviews that followed work with the calculator only on one occasion did she volunteer information about her dots. However was left until a series of follow-up interviews in January 1996 for us to begin to obtain some evidence that her imagery may be changing. When asked to think about numbers that make seven Emily's first comment was:

I just see the symbol 7 flashing in my mind waiting as if I was about to add it up... (Emily, 1995)

During our investigations into children's imagery no other low achiever had associated the word 'flashing' with symbolism to describe imagery (Pitta & Gray, 1997). The word had dominated descriptions of imagery by high achievers. Other numbers were also associated with this notion of flashing and when directly asked to talk about what she could see when she heard the word "Four" Emily responded by saying

4 flashes through my mind, and then I see, two two's like on a dice, 2 + 2, 100-96, four pounds...

Discussion

In contrast to interaction with concrete objects which requires the individual to interpret what is going on, interaction with the supercalculator offers a system in which the individual could build and test concepts first by observing and then by predicting and testing what happens. The form of presentation could be directly controlled by the child. What was becoming clear from our interactions with Emily was that she was building a different range of meanings associated with numbers and numerical symbolism – she was beginning to build a new image, a symbolic one that could stand on its own or be part of the options that would give flexibility. It seems as if her imagery was beginning to be associated with the notion of 'thought generator'.

Super calculators can carry out the evaluation of numerical expressions whilst the child can concentrate on the meaning of the symbolism that remains evident throughout. The evidence would seem to indicate that if practical activities dually focus on the process of evaluation and the meaning of the symbolism they may offer a way into arithmetic that helps those children who are experiencing difficulty develop a more powerful understanding of symbols. However, belated emphasis on the ambiguous meaning of symbolism, when the greater proportion of previous experience has emphasised procedural and manipulative aspects, is embraced with difficulty. We may need to reappraise our purpose in emphasising counting procedures with the "low achievers". It may be too late once the die is cast.

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