A Response to Werner Liedtke

Our analysis of simple arithmetic shows the way in which processes of computation and objects of mental manipulation can be represented by notation that stands ambiguously and dually for both. During the discussion we see how the young child's conception of arithmetic goes through several stages of compression; from lengthy counting procedures to concepts of arithmetic.

Werner Liedke asks whether or not we believe that the arithmetical abilities of the students can be attributable to teaching, and more importantly whether or not teachers can foster the development of number and operation sense and the growth of thinking strategies. The short answer to both questions must be "yes" but we should be quite clear that teacher's efforts to positively encourage the latter may in reality be a catalyst for the development of procedural rather than flexible thinking.

We believe our theory helps us to realise that what we need to do is to help children improve in their arithmetic is to concentrate on that flexibility. In one sense we are advocating very little more than Thornton (1978) but we believe we have an added dimension that brings caution to the belief that such an approach solves many of the difficulties we may see in the classroom.

Within the context of simple arithmetic flexibility at its highest level of efficiency is only utilised if number facts themselves are also known. However one of the problems with young children, frequently supported with their teachers responses to difficulties, is that they spend so much of their early arithmetic life counting. If counting becomes the focus of attention this can makes it very difficult to compress the number concept into an object which flexibly moves from process to object and back again. Our theory may not be the answer to all of our problems but it moves our focus of attention from the practice of procedures to the compression of those procedures. It focuses away from giving children a great deal of practice at arithmetic, at a time when that practice may cement the very counting procedures that eventually leave them to failure, and focuses the attention much more on the flexible use of number facts.

One strategy that appears to help is the provision of calculators, particularly graphic calculators at the appropriate time; no counting need to be involved when the child looks for a solution to 5+ 3. With a graphic calculator the number triple 5, 3, 8 in any combination at the same time. The reason for this is that if a child carries out a sum such as 5+3 = 8 there is no counting involved. Thus in using a calculator the child can carry out the arithmetic processes by a sequence of strokes, as opposed to carrying out counting activities. Of course this in itself will not help the child learn number facts but it would be advantageous for the child to be able to relate similar problems and observe the patterns, for example that not only is 3+4 equal to 7 but also that 4+3 is 7, 2+5 is 7 and so on. The child might try 13+4 or 23+3 and begin to see a pattern. We can help the child appreciate the pattern and develop some flexibly to solve harder combinations.

Will this strategy improve all children's ability at arithmetic? In some senses there may be a problem. After all if some children, because of their lack of success at arithmetic, turn into using procedures then what we want to do is to make them flexible, it may be that the best we can do is to teach them procedures and make them flexible. In other words we don't make them really flexible at all. What our research has done is to highlight the nature of the problem. Simply giving procedural children more examples to practice to get right may help them in one way; it may make the procedures that they use a little faster and perhaps a little more efficient, but in other ways it could be very damaging in that what they do without guidance is to develop their own idiosyncratic methods which in fact make the mathematics far harder. We therefore need to combine the practice of those facts which are essential building blocks in the system with the flexible means by which they can be manipulated most easily. This means the practice of number combinations–which eventually may include knowledge of the multiplication tables–so that they become automatic. However, this must not set arithmetic in the context of something which must be learned by rote. Those who are successful at arithmetic have more than this. They use what they know in a flexible way. The arithmetical symbol is both a process which enables them to do mathematics and a mental concept which enables them to think about it