DESCRIPTIVE AND RELATIONAL IMAGES: A FRAMEWORK FOR UNDERSTANDING NUMERICAL ACHIEVEMENT

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ABSTRACT

This paper examines the different kinds of mental image project by children at opposite ends of the achievement spectrum in elementary arithmetic. It reports the use of verbally presented concrete and abstract nouns as cues to the projection of mental images. Drawing upon psychological evidence, it presents an outline of children's disposition to qualitatively different kinds of imagery. In the arithmetical context it is not suggested that that any one kind of mental image is better than another but simply that one form allows us to do things that maybe the other does not. Our conclusions suggest that those predisposed towards the imagery of a descriptive kind focus upon an arithmetic manifest through knowing what to do. Those predisposed toward mental representations of the relational kind have 'generic' and 'proceptual' platforms that support the abstraction of the intrinsic similarities existing in numerical concepts.

INTRODUCTION

A fundamental tenet of the constructivist philosophy is that the knowledge and skills that a child brings to a given learning situation can influence the meanings that they construct in that situation. In our view such a perspective must go hand-in-hand with the additional belief that such meaning is also affected by the learners cognitive style. This, we suggest, influences what it is the learner selects from the knowledge base, how it is stored in the long-term memory and how it is used in a new situation. It is from activities associated with this knowledge base that, in the context of developing number sense, the child is expected to create mental representations which are a platform for further development.

We see cognitive development linked with the notion of number sense associated with fundamental cognitive abilities. These abilities arise from perception of the world, actions upon it and refection on both perception and action (Gray, Pitta, Pinto & Tall, 1999). However, the complexity of any activity involving objects may cause the individual to focus only on part of the activity - it is possible to focus on the objects, on the actions or a combination of the two.

Successive concept development in arithmetic involves cognitive reconstruction of the tasks engaged in. Different cognitive styles seem to indicate that differing perceptions of these tasks leads to different consequences, one associated with performing mathematical computations, the other associated with knowing mathematical concepts. Why should this be so? This paper tries to provide an answer by considering the different kinds of mental imagery projected by children . One of the guiding principles in our work is that we do not wish to be side-tracked by the visual/propositional debate (see Pavio, 1986). Irrespective of the format of mental images we believe other aspects of mental imagery that requires further discussion:

(i) the content of the mental image - which object and/or action are the focus of attention
(ii) subsequent development
(iii) the association between imagery and arithmetical achievement.

Some of our work associated with the first of these issues has been repor ed elsewhere (see for example, Gray, Pitta & Tall, 1997). The second of the issues is still the subject of investigation. It is the third that we consider in this paper to expand upon ideas devel oped within Gray and Pitta ((1995) to present a descriptive and relational aspect. Our conclusions suggest that the mental images projected by 'low achievers' are of a descri ptive kind those of 'high achievers' may be descriptive or relational.

MENTAL IMAGERY

In the current context the term image may be seen as a mental reference which is a product of imaging in any modality whether it be visual, verbal, olfactory, auditory or kinesthetic. Images may be seen as significant components of cognition (Kosslyn, 1994), they may very in abstraction, from mental analogues of a real object to a linguistic description or a symbol (Paivio, 1986). We believe that this distinction has particular relevance to the mental representations associated with cognitive development in elementary arithmetic. Tall (1995) has described how this is grounded in successive process-to-concept encapsulations, We suggest that the quality of the mental representation associated with this process may hinder or support cognitive reconstruction embraced by the notions of encapsulation or reification.

Mental Imagery and Mathematics

Within the field of mathematics it has been argued that imagery plays an important role in mathematical thinking (Lean & Clements, 1981) . Identifying images as "internally perceived holistic representations of objects that are isomorphic to their referents" (p.444), they suggest that they are mentally changed by continuous transformations corresponding to physical transformations. However, in the field of mathematics education, there does not seem to be an overall consensus about the use of terms:

> "A single term like "visual image" may have different meaning if we take it from different authors. Such an apparent mess is merely a reflection of the diversity of areas where visualisation is considered relevant and the variety of specialists who are interested in it. (Guttierez, 1996, p.4)

Irrespective of such difficulties, the role of mental imagery in the context of teaching and learning of mathematics has been a fruitful area of research. For example Dehaene & Cohen, (1994) suggest that the relationship between different forms of mental representation associated with number sense may be seen through the presentation and solution of arithmetical facts. Thomas, Mulligan and Goldin (1996) suggest that children's internal systems number representation go through a series of changes from a semiotic one in which meaning is established through previously constructed representations to an autonomous one where a new system of representation functions independently of its precursor.

Pirie and Kiernon (1994) suggest that the possession of an image, 'image having' implies that the individual does not need actions or the specific instances of image making. However, presenting a note of caution, they suggest:

> "Image having is the level at which the learners actually have some images for concept and thus they no longer need to rely on the actions that occasioned the understanding and can carry and use the ideas they have constructed. This does not imply, however, that their images are complete, appropriate or even sufficient for the work in hand. Many learners develop strong early attachments to particular dominant images and this can seriously hamper learning (Pirie & Kieran, 1994, p. 247)

Different Kinds of mental Image

Evidence, such as that given above, can not be viewed in isolation from the wider field of research on mental imagery. The work of, for example, Drake (1996) and De Beni and Pazzaglia (1995) are of particular interest, since they promote the notion of different kinds of image.

Drake's (1996) somewhat seminal article, in that it attempts to break from the science-based or quantitative paradigm for studying imagery, draws attention to three levels. At level 1, common to all within her study, subjects reported very concrete, visual images, drawn from the subjects' physical world, which were regarded as a tool or programme to achieve a particular goal. Images classified as Level 2 were usually concrete and highly pictorial but they may have included an image that acted as a symbol and not a complete picture. Though primarily visual, such images could come from other modalities. Images classified as Level 3, formed from all modalities could be very abstract.

The notion that only one kind of image exists has received extensive comment within the literature. Indeed, not only may we see distinctions between memory and imagination images, and common and bizarre images, but also distinctions are made between enactive and non-active imagery (Richardson, 1980). However, we believe that the dimension added by Cornoldi, De Beni and Pra Baldi (1988) and De Beni and Pazzagallia (1995) may lead us towards a greater understanding of cognitive differences that may explain success and failure in elementary arithmetic. Classifying mental images spontaneously evoked from a single verbal cue, Cornoldi et al introduced the notions of 'general', 'specific' and 'autobiographic' images, each being identified in decreasing proportions. General images represented a concept without any reference to a particular example; specific ones represented a well-defined example of the concept without reference to a specific episode. Autobiographic images involved the subject without reference to a specific episode. De Beni and Pazzagalia refer to autobiographical images as self-referencing images and although they do not dispute the fruitfulness of their use, th ey do question their meaning. They suggest that there is a distinction between images referring to a single episode in the subject's life (episodic-autobiographic) and those that actually involve the subject without a precise episodic reference (autobiographic images).

De Beni's and Pazzagalia's classifications were extended to notions of contextualised and non-contextualised images. Although they do not distinguish between different kinds of contextualised image, they do suggest that they would appear to contribute to the encoding of distinctive, item-specific information. On the other hand non-

contextualised images contribute to item-specific and relational encoding. It is these notions of contextualised and non-contextualised that lead us, in this paper, to discuss the qualitative differences between 'descriptive' and 'relational' forms of imagery and their occurrence in children at extremes of numerical achievement.

METHOD

Sixteen children, representing extremes of numerical achievement in each of the four years - thus two at each extreme of each year-of a "typical" primary school of the English Midlands were presented with a range of verbal items of both concrete and abstract form. Numerical achievement was measured by criterion based test results available in the school and a numerical component which formed part of a larger study of which this paper is part.

The items used were all high imagery evoking nouns for example, 'dog', 'table', football' and numerical items which were more abstract in form, for example five', 'thirty-three', 'half', 'three-quarters', 'three-quarters' and 'fraction'. A modified version of the defining feature approach (Roth & Bruce, 1995) was used to gain a sense of each child's mental image. Children's responses were obtained using semi-clinical interviews. Each verbal cue was presented with the followin g instructions:

- First Response: *What is the first thing that comes to mind when you hear the word...?*
- Free Talk Response: *Talk for 30 seconds about what comes in you're mind when you hear the word.*

The development of a two part questioning process, a 'first response' and a 'free talk response', was believed to be one that would give first an opportunity to create an image and then an opportunity to enrich the initial response with greater detail provided from a network of other relationships. Drake (1996) summarises the issues associated with this notion:

> "The generation of an image promotes the development of a trace in the brain that integrates the separate components. Thus, accessing a part for the information encoded in the memory prompts the retrieval of all the other pieces of information contained in the image" (Drake, 1996, p.7)

RESULTS

Children were interviewed over two separate occasions approximately 8 weeks apart. All response were videotaped and supported by field notes. The results are established from analysis of the video transcriptions.

Classifying the Responses

The contextual emphasis that De Beni and Pazzaglia (1995) gave to some responses within their study suggest to us that such images have descriptive and/or relational characteristics. Their finer analysis through which contextualised mental representation were classified, 'item specific' and 'relational', did not satisfy the clear distinctions observed between the responses of the subjects within the current study. Contextualised representations could describe a scene or a sequence of scenes or have

a higher order quality. Therefore the notion of an 'episodic' mental image was identified when the mental representations was associated with a description, a scene or a sequence of scenes. Representations of this form were most often narrated in continuous speech but it has to be made clear that 'episodic' mental representations are not created in the episodic memory. On the other hand, 'autobiographic-episodic' mental representations are a result of autobiographic episodic memory - they are a special case of specific examples that are enlarged through the addition of the self-schema (Kosslyn, 1994) They denote a specific scene that occurred in the individual's life. However, there were other responses that implied the existence of a context but the structure of the responses was more fragmentary; a collection of disconnected, seemingly arbitrary, generic statements which originated from the same general concepts. These seemed to serve as an anchor for expansion of the mental representation in a relational way (see Skemp (1976). Thus the notion of 'generic' representation was included. In a similar way, to identify deeper, more proceptually oriented mental representations unique to the numerical items the classification proceptual' was added.

The following different kinds of mental imagery were identified: the examples refer to first a non-numerical item (ball) and then a numerical one (fraction) :

General : the imaginistic representation of a concept without reference to a particular example of it:

"I saw a picture of a ball"
"Part of"

Specific : De Beni's reference to one well-defined example of the concept was extended to allow for multiple examples that were qualitatively similar.

"Basketball, football, tennis ball"
"Lots of different fractions for example 1/8, 1/7"

Episodic: imaginistic representations associated with some scene or sequence of scenes that were most often narrated in

"Boys can kick it around and sometimes it can get lost over the field"

"It's like doing maths and being taught how to do fractions".

Autobiographic-episodic:

category, which allowed for the "occurrence of a single episode in the subject's life connected to the concept"

"I remember my friend asking me if I wanted to play volley ball."
"My friend wasn't good at fractions and last week she had to take extra work home"

Generic these mental representations originated from the same general concept that served as the basis for explicit relational connections. They were not descriptions of a sequential event with a clear beginning and end but more often a collection of statements that seem to have the

potential to produce new ideas. Though these representations had a 'general' quality, the statements diverged to produce different ideas.

"Keeps you fit, having enjoyment, children and adults play with it, different kinds of balls"
"Shapes, part of shapes, cutting, cut shapes, sharing"

Proceptual: these imaginistic representations were identified from those that possessed a proceptual nature.

"Fractions, divisions, way of dividing things, decimals"

Analysis of Results

Here we present results relating to the 'first' and 'free talk' responses. In the 'first response' each child has given one kind of image whereas in the 'free talk response' children may have given a sequence of responses that embrace different kinds of images. All responses from each child were classified. Figures 1, 2, 3, 4 display the different kinds of images recorded. To provide a clearer sense of the more dominant mental representations classifications identified in less than 8% of instances are collected in the category 'other'.

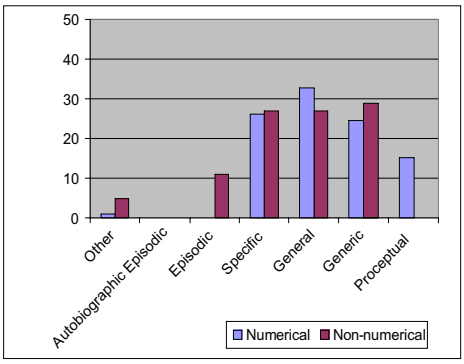


Fig 1: High Achievers' First Response

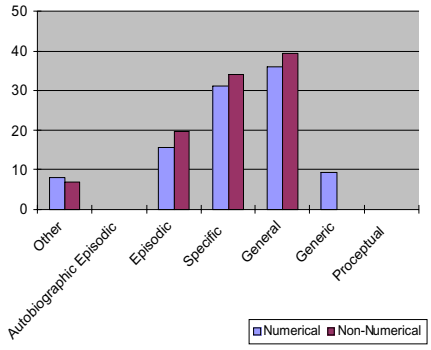


Fig 2: Low Achievers' First Response

Figures 1 and 2 provide a summary of the first response children had given to the items. For both numerical and non-numerical items general and specific mental images dominate. The essential difference observed between the two groups of children is the tendency for those high achievers' who do not project specific or general images to project generic or proceptual ones. Over 30% of the children project the former for non-numerical items, over 50% project either generic or proceptual for the numeric ones. Such a distinction is not observed amongst the low achievers

Figure 2 and 3 summarise the images projected by the children when given 30 seconds to talk about presented. Amongst the low achievers we see that items identified as generic and proceptual fail to increase during the 30 responses even though the children are given more time to respond. We see that their responses remain of a general, specific and episodic nature although the proportion of general images has decreased in favour of the episodic and specific kind. The proceptual and generic images identified amongst high achievers suggest they have an ability to link the cues to different experiences so that intrinsic similarities may be abstracted.

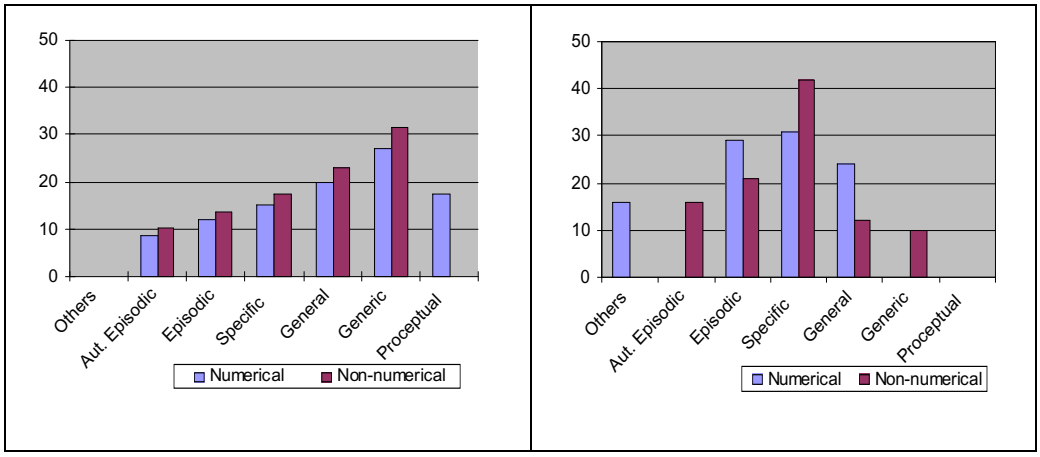


Fig: 3 High Achievers' 30 sec response

Fig: 3 Low Achievers' 30 sec response

De Beni and Pazzaglia (1995) suggest that given a short time span (10 seconds), the generation process associated with specific, autobiographic and contextual images "starts from the formation of a general skeleton-image" (p.1361). With a longer time span (40 seconds) the "image can either be enriched by more detail or, in the case of contextualised images, by the insertion of the imagined noun within a network of relationships with other objects" (Cornoldi, De Beni & Pra Baldi, 1988, p.1364). On the other hand "Episodic autobiographic images seem to have a completely different case, not only because of the used memory component (episodic-autobiographic) but also because of their generation process. The latter is not an enrichment of the general image, but a different process from the beginning: given the verbal cue, a search takes place among the autobiographic memories related to the cue, leading to the choice of the one considered the most representative" (pp. 1361-1362)

Our data confirms this but we would also suggest that apart from the specific and episodic mental images, which collectively we would call "descriptive", there are two more categories of mental images, those of a generic and proceptual kind which carry more "relational" characteristics. If we view the data from this perspective some interesting conclusions may be seen. To do this we will collapse the separated kinds of imagery into three groupings identified as 'general' images, 'descriptive' images (an amalgam of specific and episodic images) and 'relational' images (and amalgam of generic and proceptual images). We will omit the autobiographic images from this discussion since they are the result of a different generation process.

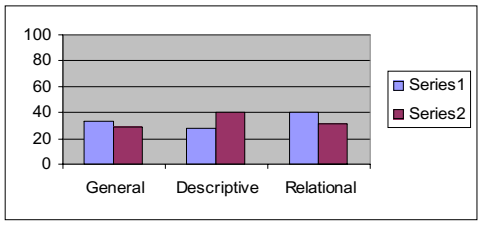


Fig. 5: Collapsed groupings of High Achievers'

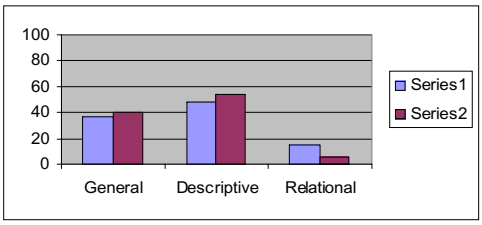
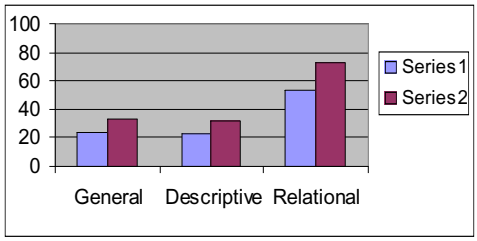
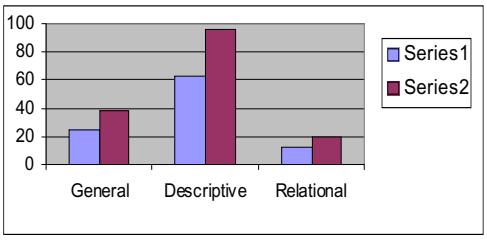


Fig. 5: Collapsed groupings of Low Achievers'

Figures 4 and 5 provide a summary of these collapsed groupings when identified for the children's first responses. The dominance of the general and relational mental images amongst high achievers is in striking when compared to the general/descriptive dominance of the low achievers. But this picture takes on a more significant perspective when the 30-second responses are collapsed as seen in figures 5 and 6.

The distinction between the high achievers and the low achievers is clearly seen when we examine the proportion of relational images projected by the two groups. Indeed, though high achievers do project both general and descriptive images, it is in the numerical context that differences are sharpest - high achievers are relational, low achievers are descriptive





Collapsed 30 secend Responses

Fig. 7: High Achievers

Fig. 8: Low Achievers.

It may be conjectured that once high achievers are given more time they tend to enlarge their concept network by involving abstract, arbitrary relationship with other objects. On the other hand images projected by the low achievers, become more specific, embellished with more detail or with the insertion of other items. These items, in contrast to those of the high achievers, expand in a sequential fashion a scene or sequence of scenes which moves like a film in the individual's minds' eyes or as a story in his mind's ears.

DISCUSSION

This study supports the notion that different kinds of imagery may be identified amongst children of elementary school age. The qualitative evidence suggests that mental representation may be either 'descriptive' or 'relational'. Those that are descriptive embrace those kinds of mental representation which project 'episodic' and 'specific' characteristics. Those that are relational project 'generic' and 'proceptual' characteristics. It is conjectured that the 'general' representation links the two in a pivotal way. It triggers the descriptive representations associated with the name, or it

triggers relational links. This difference seems to clearly exist between the 'low achievers' and the 'high achievers' and it is conjectured that the predisposition towards the one or the other is manifest in the procedural and proceptual differences that are identified in the children. We suggest that those predisposed towards the descriptive focus upon an arithmetic manifest through knowing what to do. Those predisposed toward mental representations of the relational kind have 'generic' and 'proceptual' platforms, which support the abstraction of the intrinsic similarities that, exist in numerical concepts.

However, the notion of predisposition may be interpreted in the same way as the two extremes of a spectrum. In this context nothing could be further from the truth. It is not regarded that any one kind of mental image is better than another, but simply that one form allows us to do things that maybe the other does not. 'Episodic' and 'specific' mental representations support the communication of descriptive elements. In global terms they provide something to talk about. In arithmetical terms it is conjectured that they generate things to do. The more relational mental representations provide the basis for recognising intrinsic qualities through which we may form connections between items. In arithmetical terms it is conjectured that these support the development of transformations that build upon these qualities to provide alternatives which support success.

It would therefore seem most advantageous to be able to form mental representations, which are compatible in a descriptive and relational way. At any moment one kind may be filtered out to enable us to concentrate on the other. It is conjectured that this is exactly what is happening when high achievers deal with the arithmetic. They demonstrate that throughout the range of items that formed the focus for discussion their specific and episodic mental images were similar to those of the low achievers but as we moved on to numerical items they could offer more. Low achievers do not appear to filter, they treat everything at the same level and in an arithmetical context the availability of interrelated kinds of mental image does not seem to be available to them.

It is suggested that the tendency of 'low achievers' to project mental representations of an episodic and specific kind is one of the reasons why they concentrate upon the procedural aspects of arithmetic. Their re-enactment of a process approach contrast starkly with the 'search and retrieve' approach used by high achievers. This would appear to be a quality of thinking that is more associated with selective spontaneity and not the re-enactment of an episode that needs to be specified.

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